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Preface of the editors

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In a more or less definite manner and no matter what verbal expressions might have been used, the concept that we nowadays designate by the word 'probability' must have acquired form in the mind of human beings since the dawn of thought, as a nuance added to the idea of hazard (randomness) or impredictability : hazard, but not entirely. And starting from some later time it has been remarked that what we now qualify as 'statistical' and 'statistically stable' draws away from 'hazard' and brings closer to something else that has been baptized 'probability' and has been fuzzily conceived as being virtual and 'ideal' in some sense. For at any time throughout history it has been felt that unpredictability can be more or less radical and that one may measure it by probabilities.

During a very long time the idea tied with the word 'probability' has conserved this nascent conceptual status of just an unrealized potentiality spread around the concepts of radical unpredictability — 'hazard' — and partial predictability (what we now call 'statistical'), moving without a definite contour along the dimension vaguely sketched out by these two concepts inside the magma of current, ever changing thought.

Only starting from the 17$^{\text{th}}$ century did this idea begin to acquire some inner own structure, first via a work of Blaise Pascal (1654) but mainly by Jacob Bernoulli's well known concept of *'law' of large numbers* (1690, published in 1715). Later Richard von Mises (1883 — 1953) clarified further the very peculiar relation, inside this 'law', between a just posited real number called the probability of an event from a given collection of events, and the sequence of rational numbers that express the evolving relative frequencies of the outcomes of this event when the involved random experiment is repeated: In the view of Bernoulli and of von Mises these relative frequencies *determine* — but in an ideal non effective sense — the numerical value of the probability of the considered event. Finally, in 1933 Kolmogorov endowed us with a genuine mathematical syntax of the concept of probability. Inside the general mathematical theory of measure, he has fully worked out a particular syntactic entity that he named a *probability space* founded upon *a universe of elementary events* on which an *algebra of events* is defined, on which a *probability 'measure'* is posited to 'exist': thus everything becomes both subtle and entirely definite from a formal point of view. This mathematical syntax, however, is conceived in the terms of Set Theory, whereby, likely in order to insure maximal generality, it introduces minimal logical and semantic specifications.

In this way came into being a paradigmatic case of the problem — that is far from having been solved — of *the optimality, for a given pragmatic aim, of the relations between semantic and syntax.*

This problem however was ignored, and it continues being ignored. The probabilistic syntax constructed by Kolmogorov has been considered to be able to host and to quite satisfactorily organize formally any particular *factual* probabilistic problem. Furthermore nearly nobody seemed to be troubled by the fact that, in any given factual probabilistic situation, in order to calculate predictions one has to specify numerically the individual probability of each event involved in that *particular* factual situation, whereas Kolmogorov's theory of probabilities contains exclusively *general* constraints on a probability measure, quite independent of any particular probabilistic situation.

But progressively — with remarkable slowness — under the pressure of the requirements of effectiveness that stem from the theory of computation, it became clear that the factual concept of probability inherited from Bernoulli and von Mises , incorporated in Kolmogorov's formalization just as it stands, is not an effective concept. And so it became disturbing that up to this very day no general effective method is available for defining the distribution of the numerical probabilities of the events involved in a given particular factual probabilistic situation. The strongest reaction was that of Kolmogorov himself. During the decade 1980 he kept asserting that his 'theory of probabilities' has to be considered as, exclusively, a chapter of the mathematical theory of measure, devoid of factual applicability.

On the other hand, with collaborators, Kolmogorov developed an algorithmic representation of complexities.

Furthermore various considerations and uses on a concept of randomness were developed. But concerning this concept, like in the case of the concept of probability, no clear consensus exists as yet.

Meanwhile, as it is well known, the quantum mechanical concept of probability exhibited structural specificities that hinder its incorporation in Kolmogorov's classical formal concept. And recently, several authors tried to identify the source from which the quantum probabilities could be *derived* instead of being postulated, but without stressing the distinction between factual data and formal ones. This amounts to a complexified version of the questions raised by the classical concept of probability.

In these circumstances it seemed useful to dedicate a special issue of MSCS susceptible to bring forth a *critical* and *constructive* examination of the present conceptual situation in the field of randomness-statistic-probabilities. In order to favour the emergence of such a result we called for any contribution making use of the concepts of hazard, statistic and probability, in several domains; at the same time we announced the possibility of a final debate permitting to draw global conclusions on the general nowadays concept of probability, to be published at the end of this special issue.

We warmly thank the contributors for their collaboration and we hope that this volume contains the germs of an improved and quite general concept of probability.

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Mioara Mugur-Schächter (Guest editor of this special issue of MSCS)